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Divide $f(x)$ by $(x-\alpha)^2+\beta^2$

Let $Q(x)$ be the quotient and $Ax+B$ be the reminder.

then

$$f(x) = \{(x-\alpha)^2+\beta^2\}\, Q(x) + Ax + B$$

$$= [\{x-(\alpha+i\beta)\}\{x-(\alpha-i\beta)\}]\, Q(x) + Ax + B$$

$$\Rightarrow f(\alpha+i\beta) = 0 + A(\alpha+i\beta) + B$$

$$= A(\alpha+i\beta) + B$$

$$= (A\alpha+B) + iA\beta$$

But $f(\alpha+i\beta) = 0$

equating read and imaginary parts, we see that $A\alpha+B=0$ & $A\beta=0$

But $\beta \neq 0 \Rightarrow A=0$ and so $B=0$

$\Rightarrow$ The remainder $Ax+B$ is zero.

Theorem: Every polynomial of degree n has n and only n zeroes.

Proof: Let $f(x) = a_0x^n + a_1x^{n-1} + \cdots + a_n$ where $a_0 \neq 0$, be a polynomial of degree $n \geq 1$.

By fundamental theorem of algebra, $f(x)$ has at least one zero. Let α_1 be that zero. Then $(x-\alpha)$ is a factor of $f(x)$.

Therefore we write

$$f(x) = (x-\alpha_1)\,Q_1(x), \text{ where } Q_1(x)$$

is a polynomial function of degree $n-1$.

If $n-1 \geq 1$, again by Fundamental theorem of algebra, $Q_1(x)$ has at least one zero, say α_2.

Therefore $f(x) = (x-\alpha_1)(x-\alpha_2)\,Q_2(x)$. where $Q_2(x)$ is polynomial function of degree $n-2$.

Repeating the above arguments, we get

$f(x) = (x-\alpha_1)(x-\alpha_2) \cdots (x-\alpha_n)\, Q_n(x)$

where $Q_n(x)$ is a polynomial function of degree $n-n=0$, ie $Q_n(x)$ is a constant.

Equating the coefficient of x^n on both sides of the above equation, we get

$Q_n(x) = a_0$

therefore $f(x) = a_0(x-\alpha_1)(x-\alpha_2) \cdots (x-\alpha_n)$

If α is any number other than $\alpha_1, \alpha_2, \dots \alpha_n$, then

$f(\alpha) \neq 0 \Rightarrow \alpha$ is not a zero of $f(x)$.

Hence $f(x)$ has n and only n zeros, namely $\alpha_1, \alpha_2, \dots \alpha_n$.

Hence Proved.

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ie $[\{x-(\alpha+i\beta)\}\{x-(\alpha-i\beta)\}]$ is a factor of $f(x)$.

ie, $\alpha - i\beta$ is a root of $f(x) = 0$.

Note: Let $f(x) = a_0x^n + a_1x^{n-1} + \dots + a_n$, $a_0 \neq 0$ be an nth degree polynomial in x.

Then,

$$a_0x^n + a_1x^{n-1} + \dots + a_n = 0 \quad \text{--- (1)}$$

is called a polynomial equation in x of degree n.

A number α is called a root of the equation (1) if α is a zero of the polynomial $f(x)$.

Theorem: If the equation $a_0x^n + a_1x^{n-1} + \dots + a_n = 0$ where $a_0, a_1, \dots a_n$ are real numbers ($a_0 \neq 0$) has a complex root $\alpha + i\beta$, then it also has a complex root $\alpha - i\beta$ i.e. {complex roots occur in conjugate pair for a polynomial equation with real coefficients.}

Proof: Let $f(x) = a_0x^n + a_1x^{n-1} + \dots + a_n$, $a_0 \neq$

Given that $\alpha + i\beta$ is a root of $f(x) = 0$

consider $\{x - (\alpha + i\beta)\}\{x - (\alpha - i\beta)\} = (x-\alpha)^2 +$

Paper I

Theory of equations

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Polynomial Functions:

Definition: A function defined by $f(x) = a_0x^n + a_1x^{n-1} + \cdots + a_n$, where $a_0 \neq 0$, n is a non-negative integer and $a_i\ (i=0,1,\cdots n)$ are fixed complex numbers is called a polynomial of degree n in x. Then numbers a_i are called the coefficient of f.

If α is a complex number such that $f(\alpha) = 0$, then α is called zero of the polynomial.

Fundamental theorem of algebra.

Every polynomial function of degree $n \geq 1$ has at least one zero.

Remark: Fundamental theorem of algebra says that, if $f(x) = a_0x^n + a_1x^{n-1} + \cdots + a_n$ where $a_0 \neq 0$ is the given polynomial of degree $n \geq 1$, then there exist a complex number α such that

$$a_0\alpha^n + a_1\alpha^{n-1} + \cdots + a_n = 0$$