Intersection of two conics :-

To show that two conics in general meet in Four points, real or imaginary.

Let the equation of the two conics be

$S_1 \equiv a_1x^2 + 2h_1xy + b_1y^2 + 2g_1x + 2f_1y + c_1 = 0$ — (1)

$S_2 = a_2x^2 + 2h_2xy + b_2y^2 + 2g_2x + 2f_2y + c_2 = 0$ — (2)

since both the equation are of second degree, therfore on solving one of these equations for y in term of x and substituting in the other equation.

we get an equation of the fourth degree in x giving four values of x, real or imaginary.

If we eliminate y^2 between the equations we obtain y in term of x and x^2

which shows that corresponding to one value of x, there will be one value of y.

similarly by eliminating x from (1) and (2) a biquadratic in y can also be obtained which gives four value of y real or imaginary and their corresponding value of x can be determined.

To Find the equation of the conic sections passing through the intersection of a conic and two given straight lines.

Let the equation of the conic section be

$$S \equiv ax^2+by^2+2hxy+2gx+2fy+c=0 \quad -(1)$$

and the equations of the straight lines be

$$u_1 \equiv l_1x+m_1y+n_1=0 \quad -(2)$$

and $$u_2 \equiv l_2x+m_2y+n_2=0 \quad -(3)$$

Conic (1) is cut by the line (2) in the points P and Q and by (3) in points R and T respectively.

The equation of a conic passing through their four points of intersection can be written as

$$S+\lambda u_1u_2=0 \quad -\boxed{4}$$

For different value of λ, we shall obtain different conics passing through these above points, Example: If $y = 2x$ be the equation of the chord of a circle $x^2+y^2 = 10x$. Find the equation of the circle of which this chord is a diameter.

sol^n: The equation of any circle passing through the ends of the chord intercepted on $y = 2x$ by the circle

$$x^2+y^2 = 10x \text{ is}$$

$$x^2+y^2 - 10x + \lambda(y-2x) = 0 \quad -(1)$$

or $x^2+y^2 - 2x(5+\lambda) + \lambda y = 0$

Co-ordinate of the centre of this circle is $\left(5+\lambda, -\frac{\lambda}{2}\right)$

These co-ordinate should satisfy the chord $y = 2x$, therefore

$$-\frac{\lambda}{2} = 2(5+\lambda)$$

or $\lambda = -4$

substituting this value of λ in (i), we get the required equation of the circle

$$x^2+y^2-2x(5-4) - 4y = 0$$

or $x^2+y^2-2x-4y = 0$

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System of conics :

To Find the General equation of the conic passing through the point of intersection of a curve and a straight line :-

soln : The general equation of the conic is

$S \equiv ax^2 + 2hxy + by^2 + 2gx + 2fy + c = 0$ — (1)

and the equation of the straight line

$u = lx + my + n = 0$ — (2)

consider the equation

$ax^2 + 2hxy + by^2 + 2gx + 2fy + c + \lambda(lx + my + n) = 0$ — (3)

This equation being of second degree represent a conic.

The co-ordinate of points which satisfy (1) and (2) also satisfy (3).

therefore the points of intersection of (1) and (2) lies on (3).

Therefore, (3) is the equation of conic passing through the point of intersection of (1) and (2)

i.e $S + \lambda u = 0$

Hence in general, two conics intersect in four points real or imaginary.

Note: Either all the value of x (or y) are real or an even number of them are imaginary.