Differential equation

For B.Sc/B.A Part II

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Equations solvable for y,

If the equation solvable for y, then it can be expressed in the form

$$y = f(x, p) \quad \text{——— (1)}$$

Differentiating both side of (1) with respect to x, we get a relation between x, p and $\frac{dp}{dx}$

ie $\frac{dy}{dx} = p = f\left(x, p, \frac{dp}{dx}\right)$ — (2)

it contains two variables x and p and hence it can be solved.

If let its solution be $\phi(x, p, c) = 0$ — (3)

then from it, solution can put in the form as (i) eliminate p between (1) and (3). and get a relation between x, y, c.

(ii) If it is difficult to eliminate p, solve (1) and (3) to obtain the value of x and y in term of p and c

ie $x = F_1(p, c)$, $y = F_2(p, c)$

ie solutions is in parametric form

Examples. (1) solve $y - 2px = f(xp^2)$, where $p \equiv \frac{dy}{dx}$

Sol^n: Given equation, $y - 2px = f(xp^2)$

or $y = 2px + f(xp^2)$ —— (1)

it is a form for solvable in y.

Differentiating (1) with respect to x, we get

$$\frac{dy}{dx} = p = 2p + 2x\frac{dp}{dx} + f'(xp^2)\left(p^2 + 2px\frac{dp}{dx}\right)$$

or, $$\left(p + 2x\frac{dp}{dx}\right) + p\left(p + 2x\frac{dp}{dx}\right) f'(xp^2) = 0$$

or $$\left(p + 2x\frac{dp}{dx}\right)\left(1 + pf'(xp^2)\right) = 0$$

For the general solution, we have

$$p + 2x\frac{dp}{dx} = 0$$

or $$2\frac{dp}{p} + \frac{dx}{x} = 0,$$

Integrating, we get

$$2\log p + \log x = \log c$$

or, $\log(p^2x) = \log c \Rightarrow p^2x = c$ or $p = \sqrt{c/x}$

putting the value of p in (1), we get required solution.