Analytical Geometry of two dimension
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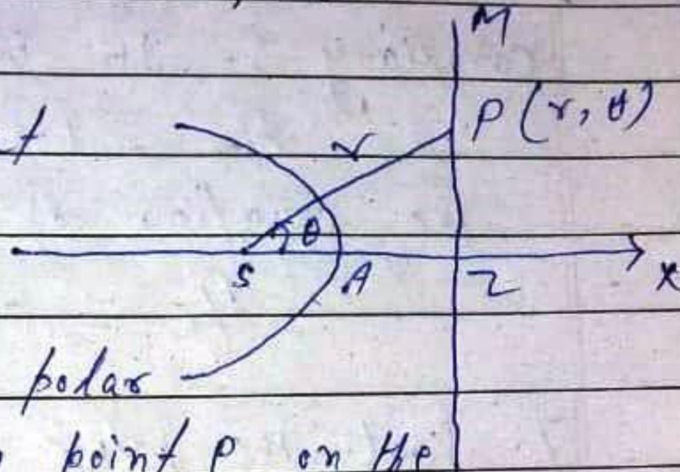
Equation to the Directrices...

To find the Polar equation of the directrix of the conic $l/r = 1 + e\cos\theta$ corresponding to the focus which is the pole.



We are to find out the equation of directrix MZ.

Let (r, θ) be the polar co-ordinates of any point P on the directrix MZ. Then $SP = r$ and $\angle PSZ = \theta$

$$\therefore \quad SZ = SP\cos\theta$$

$$\Rightarrow \quad \frac{l}{e} = r\cos\theta$$

$$\Rightarrow \quad \frac{l}{r} = e\cos\theta \quad \text{--- (1)}$$

This is the relation between the polar co-ordinates of any point on the directrix MZ, hence is the equation of the directrix MZ.

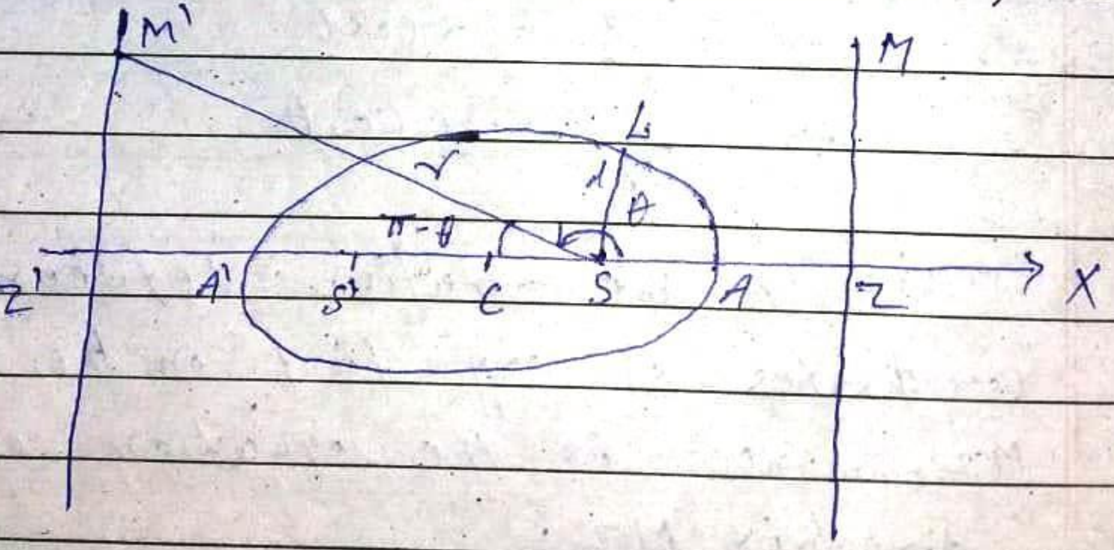
Corollary 1. If the equation of the conic is $l/r = 1 - e\cos\theta$, then the equation of the directrix MZ is

$$l/r = -e\cos\theta$$

Corollary 2. If the equation of the conic is
$l/r = 1 + e\cos(\theta - \alpha)$, then
the equation of directrix MZ is
$l/r = e\cos(\theta - \alpha)$.

Corollary 3. If the equation of the conic is
$l/r = 1 - e\cos(\theta - \alpha)$, then
the equation of directrix MZ is
$l/r = -e\cos(\theta - \alpha)$.

To find the polar equation of the directrix of the conic $l/r = 1 + e\cos\theta$ corresponding to the focus which is not the pole.



Let S and S' be the two foci of the conic $l/r = 1 + e\cos\theta$. Let ZM and Z'M' be the corresponding directrices respectively. Here one of the foci S is the pole and SZ is the initial line.

We are to find out the equation of the directrix $M'Z'$.

Let $P(r, \theta)$ be any point on $M'Z'$. Then

$$SZ' = SP\cos(\pi-\theta) = r\cos(\pi-\theta)$$
$$= -r\cos\theta$$

$$\Rightarrow ZZ' - SZ = -r\cos\theta$$

$$\Rightarrow \frac{2a}{e} - \frac{l}{e} = -r\cos\theta$$

$$\left[\because ZZ' = 2CZ' = 2\cdot\frac{a}{e} \text{ and } SZ = \frac{l}{e}\right]$$

$$\Rightarrow \frac{2a}{e} - \frac{b^2}{ae} = -r\cos\theta \quad \left[\because l = \frac{b^2}{a}\right]$$

$$\Rightarrow \frac{2a}{e} - \frac{a^2(1-e^2)}{ae} = -r\cos\theta$$
$$\left[\because b^2 = a^2(1-e^2)\right]$$

$$\Rightarrow \frac{2a}{e} - \frac{a(1-e^2)}{e} = -r\cos\theta$$

$$\Rightarrow a[2-1+e^2) = -er\cos\theta$$

$$\Rightarrow a(1+e^2) = -er\cos\theta$$

Now $l = \frac{b^2}{a} = \frac{a^2(1-e^2)}{a} = a(1-e^2)$

$$\therefore a = \frac{l}{1-e^2}$$

$$\therefore \frac{l(1+e^2)}{1-e^2} = -er\cos\theta$$

$$\Rightarrow \frac{l}{r} = -\frac{(1-e^2)}{1+e^2}\, e\cos\theta$$

which is the required equation.

Note 1. If the equation of the conic be $l/r = 1 - e\cos\theta$ then the corresponding equation of the other directrix will be

$$l/r = \left(\frac{1-e^2}{1+e^2}\right) \cdot e\cos\theta$$

Note 2. we have derived above the equations of the other directrix in case of ellipse ($e<1$). we can easily show that the same equations hold good in case of hyperbola also ($e>1$).