Also $f(0) < 0$ and $f(1) > 0$, so $f(x) = 0$ has a real roots between 0 and 1.
Therefore, the given equation must have two real roots ~~between 0 and 1~~. and two complex roots.

Example with discussion.

Consider the equation $f(x) = x^4 + 3x - 1 = 0$

This is a polynomial equations of degree 4, and hence must have four roots.

The signs of the coefficients of $f(x)$ are $+ + -$.

Therefore, the number of changes in signs $= 1$

By Descarte's rule of signs, number of real positive roots ≤ 1.

Now $f(-x) = x^4 - 3x - 1 = 0$

The signs of the coefficients of $f(-x)$ are $+ - -$; Therefore, the number of changes in sign $= 1$

Hence the number of real negative roots of $f(x) = 0$ is ≤ 1.

Therefore, the maximum number of real roots is 2.

If the equation has two real roots, then the other two roots must be complex roots. Since complex roots occur in conjugate pairs, the possibility of one real root and three complex roots is not admissible.

equation can have more positive roots than it has changes of sign.

Again, the roots of equation $f(-x)=0$ are equal to those of $f(x)=0$ but opposite to them in sign, therefore the negative roots of $f(x)=0$ are the positive roots of $f(-x)=0$ but the number of these positive roots cannot exceed the number of changes of sign in $f(-x)$; that is the number of negative roots of $f(x)=0$ cannot exceed the number of changes in sign in $f(-x)$.

All the above observation are included in the following result, known as Descarte's Rule of sign.

In any polynomial equation $f(x)=0$, the number of real positive roots cannot exceed the number of changes in the signs of the coefficients of the term in $f(x)$, and the number of real negative roots cannot exceed the number of changes in the signs of the coefficients of $f(-x)$.

continued--

Suppose that the signs of the terms in a polynomial are $++--+---+-+-$; here the number of changes of sign is 7.

We shall show that if this polynomial is multiplied by a binomial (corresponding to a positive root) whose signs are $+-$, there will be at least one more change of sign in the product than in the original polynomial.

Writing down only the sign of the terms in the multiplication, we have the following

$$
\begin{array}{ccccccccccccc}
+ & + & - & - & + & - & - & - & + & - & + & - & \\
+ & - & & & & & & & & & & & \\
\hline
+ & + & - & - & + & - & - & - & + & - & + & - & \\
 & - & - & + & + & - & + & + & + & - & + & - & + \\
\hline
+ & \pm & - & \pm & + & - & \pm & \pm & + & - & + & - & +
\end{array}
$$

Here in the last line the ambiguous sign $\pm$ is placed wherever there are two different sign to be added.

Here we see that in the product

(i) an ambiguity replaces each continuation of sign in the original polynomial;

(ii) the sign before and after an ambiguity or set of ambiguities are unlike;

(iii) a change of sign is introduced at the end:

Let us take the most unfavourable case (ie the case where the number of changes of sign is less) and suppose that all the ambiguities are replaced by continuations, then the sign of the term become

$$+ + - - + - - - + - + - +$$

and the number of changes of sign is 8.

we conclude that if a polynomial is multiplied by binomial (corresponding to a positive root) whose sign are + -, there will be at least one more change of sign in the product than in the orginal polynomial.

If then we suppose the factor corresponding to the negative and imaginary roots to be already multiplied together, each factor x-a corresponding to a positive root introduces at least one change of sign, therefore no