1

continued

Insolvability of the quintic.

A quintic function is a polynomial function of the form

$$g(x) = ax^5 + bx^4 + cx^3 + dx^2 + ex + f$$

where a, b, c, d, e, f are rational numbers, real numbers or complex numbers and a is non-zero. In other words, a quintic function is defined by a polynomial of degree five.

If a is zero but one of the coefficient b, c, d and e is non-zero, the function is classified as either a quartic function, cubic function, quadratic function or linear function.

As we noted above, solving linear, quadratic cubic and quartic equation by factorization into radicals is fairly straightforward, no matter whether the roots are rational or irrational, real or complex there are also formulae that yield the required solution. However, there are no formula for general quintic equations over the rationals in terms of radicals; this is known as the

Abel - Ruffini theorem, first published in 1824, which was one of the first application of group theory in algebra. This result also hold for equation of higher degrees.

This means that unlike quadratic cubic, and quartic polynomials, the general quintic or all polynomials of degree greater than 5 cannot be solved algebraically in term of a finite number of addition, subtractions, multiplications, divisions and root extractions.

An example quintic whose roots cannot be expressed by radicals is $x^5 - x + 1 = 0$. Some fifth-degree equations can be solved by factorizing into radicals; for example, $x^5 - x^4 - x + 1 = 0$, which can be written as $(x^2+1)(x+1)(x-1)^2 = 0$, or, as another example, $x^5 - 2 = 0$, which has $\sqrt[5]{2}$ as solution.

Evariste Galois developed techniques for determining whether a given equation could be solved by radicals which gave rise to Galois theory.

Descarte's Rule of signs and sturm's theorem.

Nature of Roots - Descarte's Rule of signs:-

To determine the nature of some of the roots of a polynomial equation it is not always necessary to solve it. For instance, the truth of the following statements will be readily admitted.

1. If the coefficients of a polynomial equation are all positive, the equation has no positive root, for example, the equation
$x^4 + 3x^2 + 3 = 0$ can not have a positive root.

2. If the coefficient of the even power of x ~~and~~ are all of one sign, and the

coefficients of the odd powers are all of the opposite sign, the equation has no negative root, thus for example the equation

$$-x^8+x^7+x^5-2x^4+x^3-3x^2+7x-3=0$$

cannot have a negative root.

3. If the equation contains only even power of x and the coefficients are all of the same sign, the equation has no real root, thus for example, the equation

$$-x^8-2x^4-3x^2-3=0$$

can not have real root.

4. If the equation contains only odd power of x and the coefficient are all of the same sign, the equation has no real root except $x=0$, thus the equation

$$x^7+x^5+3x^3+8x=0$$

has no real root except $x=0$.