Equation of conic section Referred to the Centre :-

Let the equation of the conic be

$f(x,y) = ax^2 + 2hxy + by^2 + 2gx + 2fy + C = 0$ — (1)

of which (x', y') be the centre.

Then we have,

$$ax' + hy' + g = 0 \quad \text{— (2)}$$

and, $$hx' + by' + f = 0 \quad \text{— (3)}$$

when the origin be transferred to (x', y') and axes being parallel to old one, the new equation will become,

$$ax^2 + 2hxy + by^2 + c' = 0$$

where $c' = ax'^2 + 2hx'y' + by'^2 + 2gx' + 2fy' + C$ — (4)

multiplying the equation (2) and (3) by x' and y' respectively and adding, we get

$$ax'^2 + 2hx'y' + by'^2 + gx' + fy' = 0$$

which reduces (4) in the form.

$$c' = gx' + fy' + C \quad \text{— (5)}$$

putting the values of x', y' (obtained in previous question), we get must vanish.

Therefore, $ax' + hy' + g = 0$ — (3)

$hx' + by' + f = 0$ — (4)

which on solving for x', y' gives

$$x' = \frac{hf - bg}{ab - h^2}, \quad y' = \frac{gh - af}{ab - h^2}$$

Hence, co-ordinate of the centre of the conic (1) will be:

$$\left(\frac{hf - bg}{ab - h^2}, \frac{gh - af}{ab - h^2} \right)$$

Note: If $ab - h^2 = 0$, $gh - af$ or $hf - bg = 0$

then $\frac{a}{h} = \frac{h}{b} = \frac{g}{f}$ and

the co-ordinate of the centre become meaningless ie the conic has no centre.

The above conditions with (3) and (4) give a single straight line.

In this case any point satisfying (3) or (4) may be regarded as the centre of the conic.

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Centre : To find the co-ordinates of the centre of the conic represented by general equation of second degree

$$ax^2 + 2hxy + by^2 + 2gx + 2fy + c = 0 \quad -(1)$$

Let (x', y') be the co-ordinate of the centre of the conic represented by (1).

shifting the origin at this centre and keeping the axes parallel to original axes, the equation (1) will be transformed to

$$a(x+x')^2 + 2h(x+x')(y+y') + b(y+y')^2 + 2g(x+x') + 2f(y+y') + c = 0 \quad -(2)$$

since (x', y') is the centre which has become the new origin for (2), hence any chord passing through it will be bisected at this point. this shows that if referred to new axes, (x_1, y_1) lies on (2) then $(-x_1, -y_1)$ will also lie on it as x', y' is the origin (new). since $(-x_1, -y_1)$ and (x_1, y_1) both will satisfy (2), hence it must now contain any first degree term in it.

Hence coefficients of x and y in (2)

$$c' = g\left(\frac{hf - bg}{ab - h^2}\right) + f\left(\frac{gh - af}{ab - h^2}\right) + c$$

$$= \frac{abc + 2fgh - af^2 - bg^2 - ch^2}{ab - h^2}$$

$$= \frac{\Delta}{ab - h^2}$$

Thus, the equation of the conic section referred to centre as origin, becomes

$$ax^2 + 2hxy + by^2 + \frac{\Delta}{ab - h^2} = 0$$

Note :

If, $ax^2 + 2hxy + by^2 + 2gx + 2fy + c = 0$ be the equation of the conic section, then

$$\Delta = \begin{vmatrix} a & h & g \\ h & b & f \\ g & f & c \end{vmatrix}$$

If (x', y') be the co-ordinates of the centre of conic then,

$$ax' + hy' + g = 0$$
$$hx' + by' + f = 0,$$ give the centre and

$$gx' + fy' + c = \frac{\Delta}{ab - h^2}.$$